For corresponding points,

$$d/a=m/\alpha,\ e/b=n/\beta,\ f/c=p/\gamma,\ h/a=r/\alpha,\ k/b=s/\beta,\ l/c=t/\gamma.$$

$$\therefore\ [(ma/\alpha)-h]^2+[(nb/\beta)-k]^2+[(pc/\gamma)-l]^2$$
$$=[m-(h\alpha/a)]^2+[n-(k\beta/b)]^2+[p-(l\gamma/c)]^2,$$

or $(m/\alpha-h/a)^2(a^2-\alpha^2)+(n/b-k/b)^2(b^2-\beta^2)+(p/\gamma-l/c)^2(c^2-\beta^2)=0.$

This is clearly the case since

$$h^2/a^2-k^2/b^2-l^2/c^2=m^2/\alpha^2-n^2/\beta^2-p^2/\gamma^2=1,$$

or $m^2/\alpha^2-h^2/a^2-(n^2/\beta^2-k^2/b^2)-(p^2/\gamma^2-l^2/c^2)=0.$

$$\therefore\ (m/\alpha-h/a)(m/\alpha+h/a)-(n/\beta-k/b)(n/\beta+k/b)-(p/\gamma-l/c)(p/\gamma+l/c)=0.$$

This is the case when $m/\alpha=h/a$, $n/\beta=k/b$, $p/\gamma=l/c$.

198. Proposed by JOHN J. QUINN, Professor of Mathematics, Warren High School, Warren, Pa.

Trisect an angle (1) by means of the cissoid; (2) by means of the paroboloid.

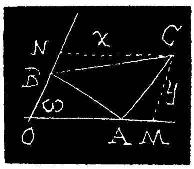
No solution of this problem has been received.

199. Proposed by F. ANDEREGG, A. M., Professor of Mathematics, Oberlin College, Oberlin, Ohio.

Two vertices of a given triangle move along fixed right lines; find the locus of the third vertex. [From Salmon's Conics, Sixth Edition, page 208, Ex. 10.]

Solution by M. W. HASKELL, Professor of Mathematics, The University of California, Berkeley, Cal.

Let us take the fixed lines as the coördinate axes, and denote the angle between them by ω; the interior angles of the triangle by A, B, C, and the lengths of the opposite sides by a, b, c, respectively; and let φ denote the variable angle OAB. From the triangles AMC, BNC, we have immediately from the theorem of sines



$$y/b=\frac{\sin(\pi-A-\varphi)}{\sin(\pi-\omega)}=\frac{\sin(A+\varphi)}{\sin\omega};\quad x/a=\frac{\sin(\omega-B+\varphi)}{\sin\omega}$$

which may immediately be rewritten in the form

$$\sin A\cos\varphi+\cos A\sin\varphi=(y/b)\sin\omega,$$
$$\sin(\omega-B)\cos\varphi+\cos(\omega-B)\sin\varphi=(x/a)\sin\omega.$$

Solving these equations for $\cos\varphi$ and $\sin\varphi$, and observing that $\sin(A+B-\omega)=\sin(C+\omega)$, we have

$$\cos\varphi\sin(C+\omega)=\sin\omega[-(x/a)\cos A+(y/b)\cos(\omega-B)]$$
$$\sin\varphi\sin(C+\omega)=\sin\omega[(x/a)\sin A-(y/b)\sin(\omega-B)].$$

Squaring and adding, and transposing the members of the equation, we obtain the required equation of the locus

$$\frac{x^2}{a^2}+\frac{2xy}{ab}\cos(C+\omega)+\frac{y^2}{b^2}=\frac{\sin^2(C+\omega)}{\sin^2\omega}.$$

The locus is therefore an ellipse with center at the intersection of the two lines.

Solved similarly by *G. W. GREENWOOD*, and *G. B. M. ZERR*.

CALCULUS.

163. Proposed by F. P. MATZ, Sc. D., Ph. D., Professor of Mathematics and Astronomy in Defiance College, Defiance, Ohio.

Can there be a plane curve the length of which varies *directly as the abscissa* and *inversely as the ordinate* of any point on the curve?

Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

$S=mx/y$, $dS=\dfrac{mydx-mxdy}{y^2}=\sqrt{(dx^2+dy^2)}$.

$\therefore\ \sqrt{[1+(dx/dy)^2]}=[my(dx/dy)-mx]/y^2$.
Let $x=vy$, $dx/dy=v+y(dv/dy)=v+yp$, suppose.
$\therefore\ \sqrt{[1+(v+yp)^2]}=mp$, or $v+yp=\sqrt{(m^2p^2-1)}$.....(1).

From (1), $p=\dfrac{vy\pm\sqrt{(v^2m^2+m^2-y^2)}}{m^2-y^2}=\dfrac{xy\pm\sqrt{[m^2(x^2+y^2)-y^4]}}{y(m^2-y^2)}$.

Differentiating (1) with respect to y,

$$dv/dy=p=p--y(dp/dy)+\frac{m^2p(dp/dy)}{\sqrt{(m^2p^2-1)}}.$$

$\therefore\ 2p=\left(\dfrac{m^2p}{\sqrt{(m^2p^2-1)}}-y\right)\dfrac{dp}{dy}$.

$\therefore\ \dfrac{dy}{dp}=\dfrac{m^2}{2\sqrt{(m^2p^2-1)}}-\dfrac{y}{2p}$ or $\dfrac{dy}{dp}+\dfrac{y}{2p}=\dfrac{m^2}{2\sqrt{(m^2p^2-1)}}$.

$\therefore\ y\sqrt{p}=C+\dfrac{m^2}{2}\displaystyle\int\frac{\sqrt{(p)}dp}{\sqrt{(m^2p^2-1)}}=C+\frac{m^2}{2}f(p)$.....(2).